Results are shown of an experimental study concerning the heat transfer in liquefied gases (nitrogen, argon, and air) in the super-critical range. The test data are generalized on the basis of thermodynamic similarity. Universal relations are derived.

The thermophysical properties of substances in the supercritical state change abruptly, which leads to some peculiarities in the heat transfer process.

The authors have studied the heat transfer in liquefied gases (nitrogen, argon, and air) under supercritical pressures on a test stand a description of which was given in [1].

The range of test temperatures was $0.87 \leq T / T_{c r} \leq 1.19$. The pressure was varied over the ranges $1.175 \leq P / P_{\text {cr }} \leq 2.92$ in nitrogen and $1.175 \leq P / P_{c r} \leq 2.04$ in argon.

Pressures higher than $P / P_{c r}=2.04$ could not be produced for argon on this stand, because the latter had been designed for an operating pressure of 100 bars.

The heat load was varied from 550 to $11,500 \mathrm{~W} / \mathrm{m}^{2}$.
The test materials were $99 \%$ and $99.999 \%$ nitrogen concentrates and grade A argon according to GOST (Government Standards) 10-157-62.

Nitrogen was tested in horizontal tubes made of grade Kh18N9T steel in two diameters: 2.80 and 10.01 mm , argon was tested in the 2.80 mm tube.

Air was tested in the 10.01 mm tube at a single heat load level $\mathrm{q}=3150 \mathrm{~W} / \mathrm{m}^{2}$ under pressures $\mathrm{P} / \mathrm{P}_{\mathrm{cr}}=1.175$ and 1.47 .


Fig. 1. Heat transfer coefficient $\alpha\left(\mathrm{W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{K}\right)$ as a function of the nitrogen temperature $\mathrm{T}_{\mathrm{N}_{2}}\left({ }^{\circ} \mathrm{K}\right)$, at q $=3000 \mathrm{~W} / \mathrm{m}^{2}: \mathrm{P} / \mathrm{P}_{\mathrm{cr}}=1.175(1), 1.47(2), 1.75(3)$.

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Fig. 2


Fig. 3

Fig. 2. Maximum heat transfer coefficient $\alpha_{\max }\left(\mathrm{W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right)$ as a function of the pressure $\mathbf{P} / \mathbf{P}_{\mathrm{cr}}$, at various heat load levels $q\left(W / \mathrm{m}^{2}\right): 550 \mathrm{~W} / \mathrm{m}^{2}(1), 3000 \mathrm{~W} / \mathrm{m}^{2}(2), 5000 \mathrm{~W} / \mathrm{m}^{2}(3), 11,200 \mathrm{~W} / \mathrm{m}^{2}$ (4).

Fig. 3. Heat transfer coefficient $\alpha\left(\mathrm{W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right)$ as a function of the argon temperature $\mathrm{T}_{\mathrm{Ar}}\left({ }^{\circ} \mathrm{K}\right)$, at $\mathrm{q}=11,500 \mathrm{~W} / \mathrm{m}^{2}: \mathrm{P} / \mathrm{P}_{\mathrm{cr}}=1.175$ (1), 1.47 (2), 1.75 (3).

The heat transfer coefficient $\alpha$ as a function of the nitrogen temperature $\mathrm{T}_{2}$ is shown in Fig. 1 for various pressures in the 2.80 mm tube.

According to the graphs, coefficient $\alpha$ passes through a peak. This peak becomes lower with rising pressure and shifts toward higher temperatures. The curve becomes flatter with rising pressure.

The results indicate that at pressures $\mathrm{P} / \mathrm{P}_{\mathrm{cr}}=2.34$ and 2.92 , far beyond the critical level, $\alpha$ has no peak and varies negligibly little with temperature. Such a trend is noted at all the given heat load levels.

This pattern of $\alpha$ variation within the test range is explained by the way in which the thermophysical properties of nitrogen depend on the temperature and the pressure. An analysis of these relation shows that $\alpha_{\max }$ corresponds to temperatures $\mathrm{T}_{\mathrm{m}}$ at which $\mathrm{c}_{\mathrm{p}}$ is maximum. These temperatures at pressures plotted in Fig. 1 are respectively $\mathrm{T}_{\mathrm{m}} \approx 129^{\circ} \mathrm{K}$ at $\mathrm{P}=38.5$ bars, $\mathrm{T}_{\mathrm{m}}=134^{\circ} \mathrm{K}$ at $\mathrm{P}=48.0 \mathrm{bars}$, and $\mathrm{T}_{\mathrm{m}}$ $=140^{\circ} \mathrm{K}$ at $\mathbf{P}=57.5$ bars.

The graphs in Fig. 1 indicate also that $\alpha$ does not depend much on the pressure at nitrogen temperatures $\mathrm{T}_{\mathrm{N}_{2}} \gg \mathrm{~T}_{\mathrm{m}}$ and $\mathrm{T}_{\mathrm{N}_{2}} \ll \mathrm{~T}_{\mathrm{m}}$.

As has been mentioned earlier [2-4], the peak value of $\alpha$ decreases with increasing heat load q. Such a trend of $\alpha_{\text {max }}$ is noted also here under pressures $\mathrm{P} / \mathrm{P}_{\mathrm{cr}}<1.4$. Under $\mathrm{P} / \mathrm{P}_{\mathrm{cr}}>1.4$ the pattern reverses (Fig. 2).

It must be added that $P / P_{\text {cr }}$ did not exceed 1.2 in the $[2-4]$ tests.
When explaining the mechanism of heat transfer in the supercritical range, Goldman [5] notes that large clusters of molecules in the liquid break up as they come in contact with the heat transfer surface and form nodules of single molecules within the liquid.

Apparently, under near-critical pressures $\mathrm{P} / \mathrm{P}_{\mathrm{cr}}<1.4$ and near $\mathrm{T}_{\mathrm{m}}$ under heavy heat loads these nodules form a layer which separates the wall from the liquid. This layer has a high thermal resistance and this results in an abruptly increasing wall temperature. As the pressure is raised further, $\mathrm{P} / \mathrm{P}_{\text {cr }}$ $>1.4$, the formation of those nodules becomes more difficult and they become less stable. For this reason, the trend of the $\alpha$ variation with the heat load reverts to normal.

TABLE 1. Values of $\mathrm{Gu}, \mathrm{Me}$, and K for Several Gases

| substance | Criterial number |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{Gu}=\frac{T_{\mathrm{s}}}{T_{\mathrm{s}}}$ | $\mathrm{Me}=\ln { }^{\text {P }} \mathrm{Cr}$ | $K=\frac{R T \mathrm{cr}}{P^{P} \mathrm{cr}{ }^{V} \mathrm{cr}}$ |
| Nitrogen | 0,612 | 3,51 | 3,42 |
| Argon | 0,580 | 3,86 | 3,43 |
| Air | 0,605 | 3,64 | 3,42 |
| Oxygen | 0,585 | 3,93 | 3,42 |
| Methane | 0,585 | 3,85 | 3,52 |
| Carbon oxide | 0,605 | 3,55 | 3,56 |

TABLE 2. Values of Exponents $\mathrm{m}_{1}, \mathrm{~m}_{2}, \mathrm{~m}_{3}$, and $\mathrm{m}_{4}$

| With $n=0,16$ |  | With $n=0,1$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $m_{1}$ | $m_{2}$ | $m_{3}$ | $m_{4}$ | $m_{1}$ | $m_{2}$ | $m_{3}$ |
| $-0,42$ | 0,42 | 0,84 | $-0,58$ | $-0,45$ | 0,45 | $m_{4}$ |

The nitrogen tests in the 10.01 mm tube have shown that the tube dimensions do not, within measuring accuracy, affect the value of $\alpha$. It is well known that in the $\operatorname{GrPr}>2 \cdot 10^{7}$ range the heat transfer process is autonomous [6]. In our tests the product $\mathrm{Gr} \operatorname{Pr}$ varied from $3 \cdot 10^{5}$ to $10^{10}$.

No effect of the impurity content on the value of $\alpha$ was noted as the concentration of nitrogen had changed from $99 \%$ to $99.999 \%$. An explanation for this may be that the impurities in nitrogen obtained from air are oxygen and argon, whose critical parameters do not differ much from those of nitrogen.

The heat transfer coefficient $\alpha$ as a function of the argon temperature under a heat load $q=11,500$ $\mathrm{W} / \mathrm{m}^{2}$ is shown in Fig. 3 for various pressures. According to the graphs, $\alpha$ here also passes through a peak as the temperature is varied.

The value of $\alpha_{\text {max }}$ decreases with rising pressure and shifts toward higher temperatures.
In the case of argon, too, one notes that $\alpha_{\text {max }}$ decreases with increasing heat load $q$ when $P / P_{c r}$ $<1.4$ and that $\alpha_{\text {max }}$ increases with increasing heat load $q$ when $P / P_{c r}>1.4$.

Thus, the trends of the $\alpha$ variation depending on the test parameters are the same for argon as for nitrogen.

Coefficient $\alpha$ is usually determined from criterial equations derived on the basis of a test data evaluation by the theory of similarity. The physical properties of substances are then considered constant.

Within certain ranges of practical interest, however, such as the supercritical range, for instance, the physical properties vary considerably and cannot be considered constant. Especially appreciable is the change in the specific heat $c_{p}$, which under normal conditions depends very little on the temperature.

This fact must be taken into consideration in establishing the similarity laws for heat transfer processes. Namely, to the system of differential equations describing these processes must be added equations


Fig. 4. Evaluation of test data in terms of thermodynamic similarity criteria: nitrogen (1), argon (2), air (3). Axis of abscissas $\left(\mathrm{T}_{\mathrm{L}} / \mathrm{T}_{\mathrm{cr}}\right) /\left(\mathrm{T}_{\mathrm{m}} / \mathrm{T}_{\mathrm{cr}}\right)$, axis of ordinates

$$
\begin{gathered}
\text { zone I: } \frac{\alpha}{q^{0.16}\left(\frac{g R}{M}\right)^{0.42} P_{\mathrm{cr}}^{0.84} T_{\mathrm{cr}}^{-0.58}\left(\frac{P}{P_{\mathrm{Cr}}}\right)^{0.2}} ; \\
\text { zones II and III: } \frac{\alpha}{q^{0.1}\left(\frac{g R}{M}\right)^{0.45} P_{\mathrm{cr}}^{0.9} T_{\mathrm{cr}}^{0.55}\left(\frac{P}{P_{\mathrm{cr}}}\right)^{-0.45}} ; \\
\text { zone IV: } \frac{\alpha}{q^{0.1}\left(\frac{g R}{M}\right)^{0.45} P_{\mathrm{Cr}}^{0.9} T_{\mathrm{cr}}^{-0.55}\left(\frac{P}{P_{\mathrm{cr}}}\right)^{0.2}}
\end{gathered}
$$

which define the temperature dependence and the pressure dependence of the physical properties. These equations may differ in form from substance to substance, since in most cases they are purely empirical. Their addition to the fundamental system of differential equations results in different complete systems for different substances.

Consequently, the phenomena under study here may not be assigned to the same class.
The criterial relations based on the theory of thermal similarity are applicable to all phenomena of any given class characterized by the same values of the similarity numbers. It follows from here that a derivation of such universal relations for the specific cases in our study is fraught with difficulties.

Universal relations of a more limited scope can be obtained on the basis of the thermodynamic similarity theory.

According to [7], any thermophysical properties of substances can be expressed as a product of two quantities: a dimensional and a dimensionless one:

$$
\begin{equation*}
\chi=f_{1}\left(\frac{M}{g}, T_{\mathrm{cr}}, P_{\mathrm{cr}}, R\right) f_{2}\left(\frac{P}{P_{\mathrm{cr}}}, \frac{T}{T_{\mathrm{cr}}}, \frac{C_{V_{\mathrm{o}}}}{R}\right) \tag{1}
\end{equation*}
$$

The relation between the fundamental quantities affecting the rate of heat transfer during boiling in a large volume can, within practical accuracy, be represented by a power law [8]:

$$
\begin{equation*}
\alpha=C\left(q^{n}, \lambda^{n_{1}}, \rho^{n_{2}}, C_{p}^{n_{3}}, \ldots,\right) \tag{2}
\end{equation*}
$$

In the case of thermodynamically similar substances, relation (2) with (1) for boiling in a large volume can be represented as [8]:

$$
\begin{equation*}
\alpha=C q^{n}\left(\frac{M}{g}\right)^{m_{1}} R^{m_{3}} P_{\mathrm{cr}}^{m_{3}} T_{\mathrm{cr}}^{m_{s}} f\left(\frac{T}{T_{\mathrm{cr}}}, \frac{P}{P_{\mathrm{cr}}}\right) . \tag{3}
\end{equation*}
$$

The rate of heat transfer during natural convection is defined in terms of the same parameters.
As has been said already, $\alpha_{\max }$ decreases with rising pressure and shifts toward higher temperatures, i.e., the variation of $\alpha$ follows a particular trend at every relative pressure level. In order to combine all these trends into a single expression, we have introduced the parameter $\left(\mathrm{T}_{\mathrm{L}} / \mathrm{T}_{\mathrm{cr}}\right) /\left(\mathrm{T}_{\mathrm{m}} / \mathrm{T}_{\mathrm{cr}}\right)$.

In this way, for our case Eq. (3) becomes

$$
\begin{equation*}
\alpha=C q^{n}\left(\frac{M}{g}\right)^{m_{\mathrm{z}}} R^{m_{\mathrm{z}}} P_{\mathrm{cr}}^{m_{\mathrm{s}}} T_{\mathrm{cr}}^{m_{\mathrm{L}}} f\left(\frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}} / \frac{T_{m}}{T_{\mathrm{cr}}}, \frac{P}{P_{\mathrm{cr}}}\right) . \tag{4}
\end{equation*}
$$

The right-hand side of this equation must have the dimension of $\alpha$. Knowing the relation between $\alpha$ and $q$ from tests, one can determine the exponents $m_{1}, m_{2}, m_{3}$, and $m_{4}$ by dimensional analysis. These exponents are then [9]

$$
m_{1}=-m_{2}=-\frac{n-1}{2} ; m_{8}=1-n ;-m_{4}=\frac{1+n}{2} .
$$

Our study was concerned with thermodynamically similar gases. The values of the criterial numbers K for nitrogen, argon, and air are close. Their Guldberg numbers Gu and Mendeleev numbers Me are close. Thermodynamically similar gases are also $\mathrm{O}_{2}, \mathrm{CH}_{4}$, and CO . The data in Table 1 confirm their approximate thermodynamic similarity.

Thus, if we generalize the test data on the basis of the thermodynamic similarity theory, then they will apply to all these gases.

Exponent $n$ of the factor $q$ in Eq. (4) has been determined from test data. In the $T_{L} / T_{m}<0.97$ range n is equal to 0.16 , in the $\mathrm{T}_{\mathrm{L}} / \mathrm{T}_{\mathrm{m}}>0.97$ range it is equal to 0.1 .

Values of the other exponent are listed in Table 2.
A further analysis has shown that the test data ought to be evaluated in terms of coordinates

$$
\frac{\alpha}{A q^{n}\left(\frac{P}{P_{\mathrm{cr}}}\right)^{t}}-f\left(\frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}} / \frac{T_{m}}{T_{\mathrm{cr}}}\right)
$$

where $A=(M / g){ }^{m_{1}} R^{m_{2}} P_{c r}^{m_{3}} T_{c r} m_{4}$.

This relation based on test data is shown in Fig. 4 for all the gases in our study.
Exponent $l$ of the $P / P_{c r}$ parameter is equal to -0.45 within the range $0.97<\left(\mathrm{T}_{\mathrm{L}} / \mathrm{T}_{\mathrm{cr}}\right) /\left(\mathrm{T}_{\mathrm{m}} / \mathrm{T}_{\mathrm{cr}}\right)$ $<1.03$ and equal to 0.2 in the range $0.97>\left(\mathrm{T}_{\mathrm{L}} / \mathrm{T}_{\mathrm{cr}}\right) /\left(\mathrm{T}_{\mathrm{m}} / \mathrm{T}_{\mathrm{cr}}\right)>1.03$.

This graph indicates four regions of different $\alpha$ variation modes. The mathematical expressions for these four regions are:

$$
\begin{gather*}
0.74 \leqslant \frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}} / \frac{T_{m}}{T_{\mathrm{cr}}} \leqslant 0.97, \\
\alpha=14.4 \frac{P_{\mathrm{cr}}^{0.84}}{M^{0.42} T_{\mathrm{cr}}^{0.58}}\left(\frac{P}{P_{\mathrm{cr}}}\right)^{0.2}\left[7.63\left(\frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}} / \frac{T_{m}}{T_{\mathrm{cr}}}\right)-1\right] q^{0.16}, \mathrm{~W} / \mathrm{m}^{2} \cdot \mathrm{deg}  \tag{5}\\
0.97 \leqslant \frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}}\left(\frac{T_{m}}{T_{\mathrm{cr}}} \leqslant 1.00,\right. \\
\begin{array}{c}
\alpha=1440 \frac{P_{\mathrm{cr}}^{0.5}}{M^{0.45} T_{\mathrm{cr}}^{0.55}}\left(\frac{P}{P_{\mathrm{cr}}}\right)^{-0.45}\left[1.124\left(\frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}} / \frac{T_{m}}{T_{\mathrm{cr}}}\right)-1\right] q^{0.1}, \\
1.00 \leqslant \frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}} / \frac{T_{m}}{T_{\mathrm{cr}}} \leqslant 1.03, \\
\alpha=3420 \frac{P_{\mathrm{cr}}^{0.9}}{M^{0.45} T_{\mathrm{cr}}^{0.55}}\left(\frac{P}{P_{\mathrm{cr}}}\right)^{-0.45}\left[1-0.948\left(\frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}} / \frac{T_{m}}{T_{\mathrm{cr}}}\right)\right] q^{0.1} \\
1.03 \leqslant \frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}}\left[\frac{T_{m}}{T_{\mathrm{cr}}} \leqslant 1.14,\right. \\
\alpha=695 \frac{P_{\mathrm{cr}}^{0.9}}{M^{0.45} T_{\mathrm{cr}}^{0.55}}\left(\frac{P}{P_{\mathrm{cr}}}\right)^{0.2}\left[1-0.858\left(\frac{T_{\mathrm{L}}}{T_{\mathrm{cr}}} / \frac{T_{m}}{T_{\mathrm{cr}}}\right)\right] q^{0.1}
\end{array} . \tag{6}
\end{gather*}
$$

Pressure $P$ in formulas (5)-(8) is measured in $N / \mathrm{cm}^{2}$. In the ranges $0.97>\left(\mathrm{T}_{\mathrm{L}} / \mathrm{T}_{\mathrm{cr}}\right) /\left(\mathrm{T}_{\mathrm{m}} / \mathrm{T}_{\mathrm{cr}}\right)$ $>1.03$ the mean deviation of test points from the averaging curve is $\pm 10 \%$, the maximum deviation is $\pm 15 \%$. Within the $0.97<\left(\mathrm{T}_{\mathrm{L}} / \mathrm{T}_{\mathrm{cr}}\right) /\left(\mathrm{T}_{\mathrm{m}} / \mathrm{T}_{\mathrm{cr}}\right)<1.03$ range the deviation of most test points does not exceed $\pm 15 \%$ and only a few values of $\alpha$ at relative pressures and temperatures $\mathrm{T}_{\mathrm{L}} / \mathrm{T}_{\mathrm{m}}$ close to unity deviate by up to $\pm 30 \%$ under $q<3000 \mathrm{~W} / \mathrm{m}^{2}$.

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